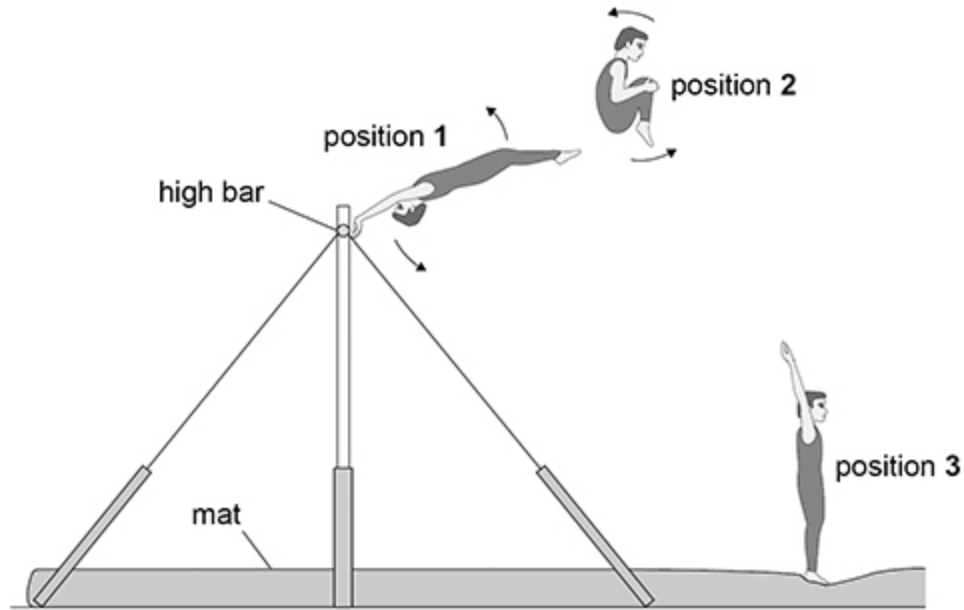


1. A gymnast dismounts from an exercise in which he swings on a high bar. The gymnast rotates in the air before landing.

The figure below shows the gymnast in three positions during the dismount.



The arrows show the direction of rotation of the gymnast.

In position **1** the gymnast has just let go of the bar. His body is fully extended.

Position **2** shows the rotating gymnast a short time later. His knees have been brought close to his chest into a 'tuck'.

Position **3** is at the end of the dismount as the gymnast lands on the mat. His body is once again fully extended.

- (a) Explain why the moment of inertia about the axis of rotation decreases when his knees are moved towards his chest.

Go on to explain the effect this has on his angular speed.

(3)

The table below gives some data about the gymnast in position 1 and in position 2.

Position	Moment of inertia / kg m ²	Angular speed / rad s ⁻¹
1	13.5	ω
2	4.1	14.2

- (b) Calculate the angular speed ω of the gymnast in position 1.

$$\omega = \text{_____ rad s}^{-1}$$

(1)

(c) The gymnast stays in the tuck for 1.2 s.

Determine the number of **complete** rotations performed by the gymnast when in the tuck during the dismount.

number of complete rotations = _____

(2)

(d) The gymnast repeats the exercise. The height of the bar remains unchanged.

State and explain **two** actions the gymnast can take to complete more rotations during the dismount.

1 _____

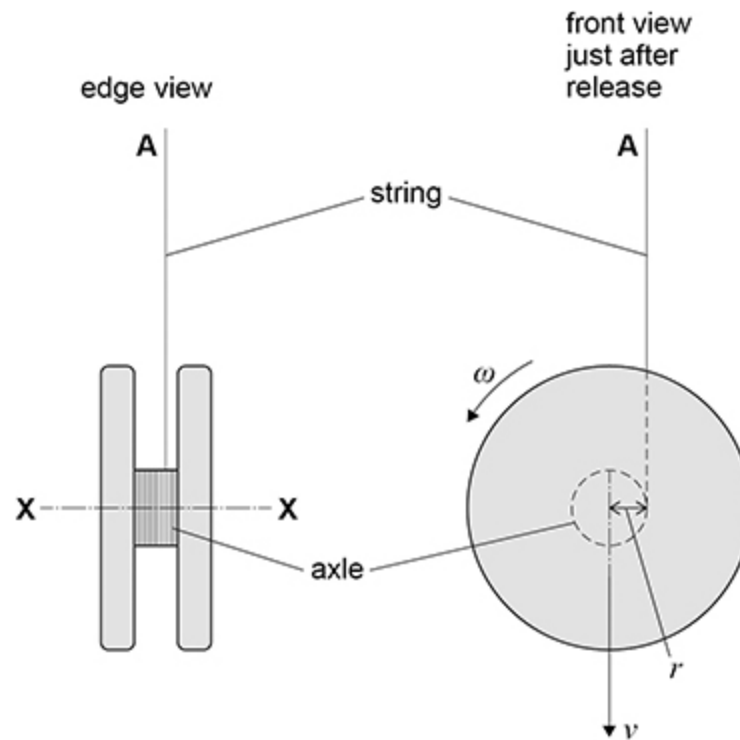
2 _____

(4)

(Total 10 marks)

2. **Figure 1** shows a yo-yo made of two discs separated by a cylindrical axle. Thin string is wrapped tightly around the axle.

Figure 1



Initially both the free end **A** of the string and the yo-yo are held stationary.

With **A** remaining stationary, the yo-yo is now released so that it falls vertically. As the yo-yo falls, the string unwinds from the axle so that the yo-yo spins about its centre of mass.

The linear velocity v of the centre of mass of the falling yo-yo is related to the angular velocity ω by $v = r\omega$ where r is the radius of the axle.

- (a) The yo-yo accelerates uniformly as it falls from rest. The string remains taut and has negligible thickness.

$$\text{mass of yo-yo} = 9.2 \times 10^{-2} \text{ kg}$$

$$\text{radius of axle} = 5.0 \times 10^{-3} \text{ m}$$

$$\text{moment of inertia of yo-yo about axis } \mathbf{X-X} = 8.6 \times 10^{-5} \text{ kg m}^2$$

When the yo-yo has fallen a distance of 0.50 m, its linear velocity is V .

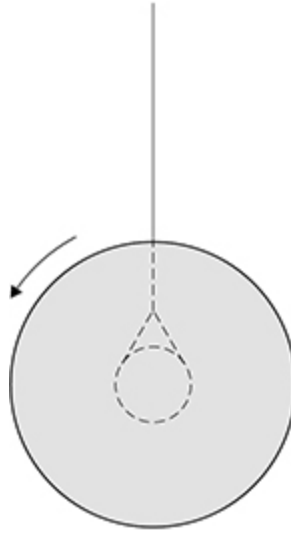
Calculate V by considering the energy transfers that occur during the fall.

$$V = \underline{\hspace{2cm}} \text{ m s}^{-1}$$

(3)

- (b) The yo-yo falls further until all the string is unwound. The yo-yo then 'sleeps'. This means the yo-yo continues to rotate in a loose loop of string as shown in **Figure 2**.

Figure 2



The string applies a constant frictional torque of 8.3×10^{-4} N m to the axle. The angular velocity of the yo-yo at the start of the sleep is 145 rad s^{-1} .

Determine, in rad, the total angle turned through by the yo-yo during the first 10 s of sleeping.

angle = _____ rad

(3)

(Total 6 marks)

3. (a) State the condition necessary so that the law of conservation of angular momentum applies to a rotating system.

(1)

A clutch is used to connect two rotating shafts together so that they rotate at the same speed.

- (b) The figure shows two shafts, **A** and **B**, rotating freely about the same axis. **Table 1** gives information about the two shafts.

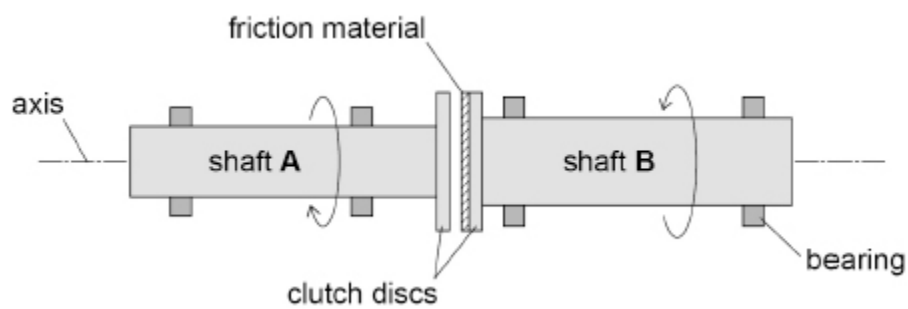


Table 1

	Shaft A	Shaft B
Angular speed / rad s^{-1}	95	45
Moment of inertia / kg m^2	7.2	11.5
Direction of rotation	Clockwise looking from left	Anticlockwise looking from left

The two shafts are connected by forcing the clutch discs together.
Friction acts between the discs and slipping occurs for a short time until both shafts rotate at a common angular speed.
The clutch is now said to be engaged.

Show that the common angular speed of the two shafts immediately after the clutch is engaged is about 9 rad s^{-1} .

State whether the direction of the common angular speed is clockwise or anticlockwise when viewed from the left.

direction when viewed from the left = _____

(3)

- (c) **Table 2** gives information about two clutches, **C** and **D**.
C and **D** provide different constant frictional torques during slipping at the clutch discs.

Table 2

Clutch	Frictional torque during slipping / N m
C	600
D	320

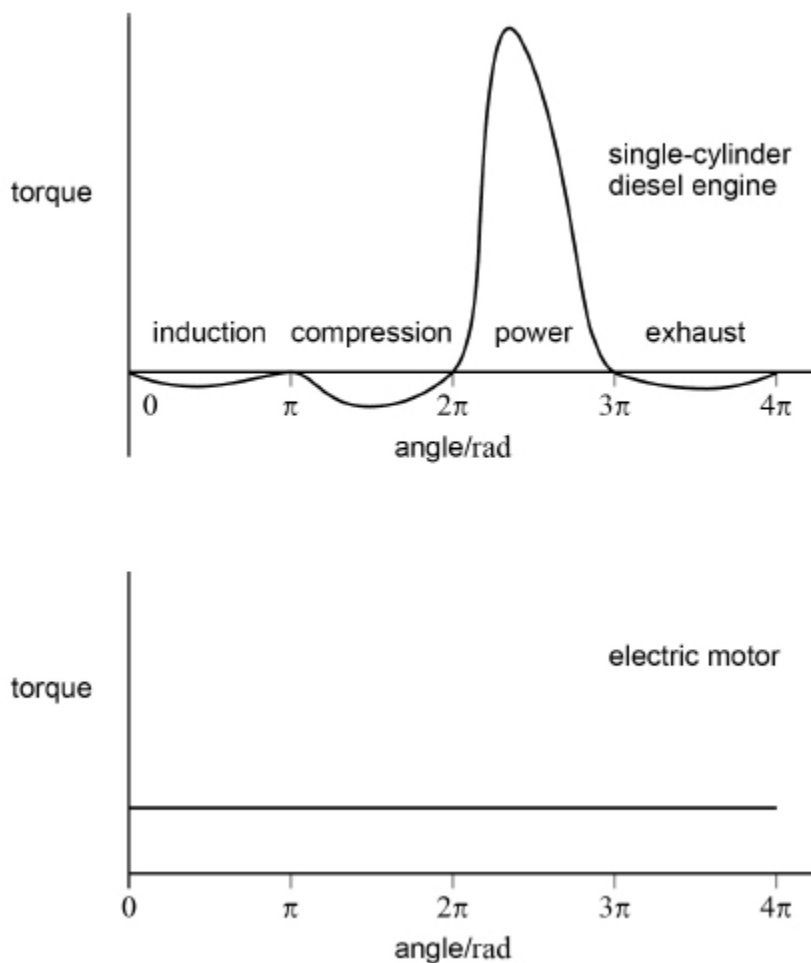
The slipping time is to be kept between 1.0 s and 2.0 s with the same initial conditions shown in **Table 1**, and the same final common angular speed.

Deduce whether either or both clutches allow this.

(3)
(Total 7 marks)

4. A turning moment diagram is a graph showing the variation of torque produced by an engine or motor with the angle of rotation of the output shaft.

The graph below shows the turning moment diagrams for a single-cylinder diesel engine and an electric motor that have the same output power.



- (a) State what is represented by the area between the curve and the angle axis for a turning moment diagram.

(1)

- (b) The diesel engine or the electric motor may be used to drive a machine that has a low moment of inertia and that requires an almost constant torque.

Discuss why, to drive this machine, the diesel engine would need to be fitted with a flywheel.

In your answer you should explain

- why the electric motor does **not** require a flywheel
- why the torque of the diesel engine varies over one cycle, including why there are points where the torque is zero
- how the moment of inertia of the flywheel influences the motion of the output shaft of the diesel engine.

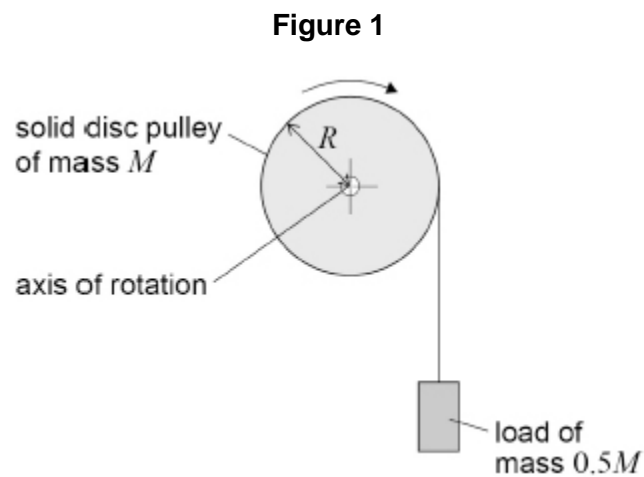
(6)
(Total 7 marks)

5.

- (a) State what is meant by the moment of inertia of an object about an axis.

(1)

- (b) A student does an experiment using the apparatus shown in **Figure 1**.



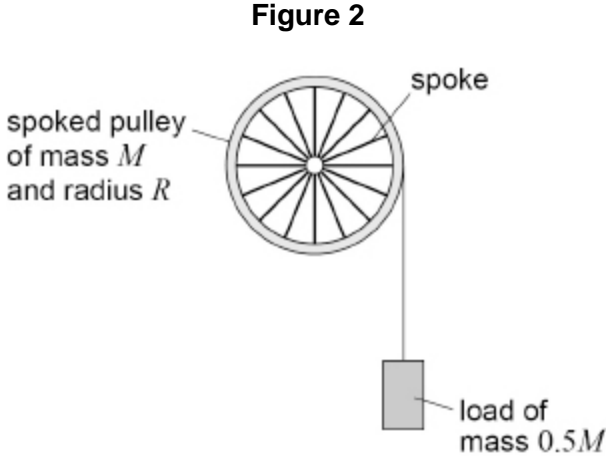
A solid disc pulley of mass M and radius R is supported in bearings which have negligible friction.

A string of negligible mass is wrapped around the circumference of the pulley. A load of mass $0.5M$ is fixed to the free end of the string. The string does not slip on the pulley. The moment of inertia of the pulley about the axis of rotation is $0.5MR^2$. The student holds the pulley stationary.

When the student releases the pulley, the load accelerates downwards uniformly and is at a velocity v after moving a distance h .

Show that the acceleration of the load is $0.5g$.

(c) The student repeats the experiment using a spoked pulley of the same mass and radius, as shown in **Figure 2**.



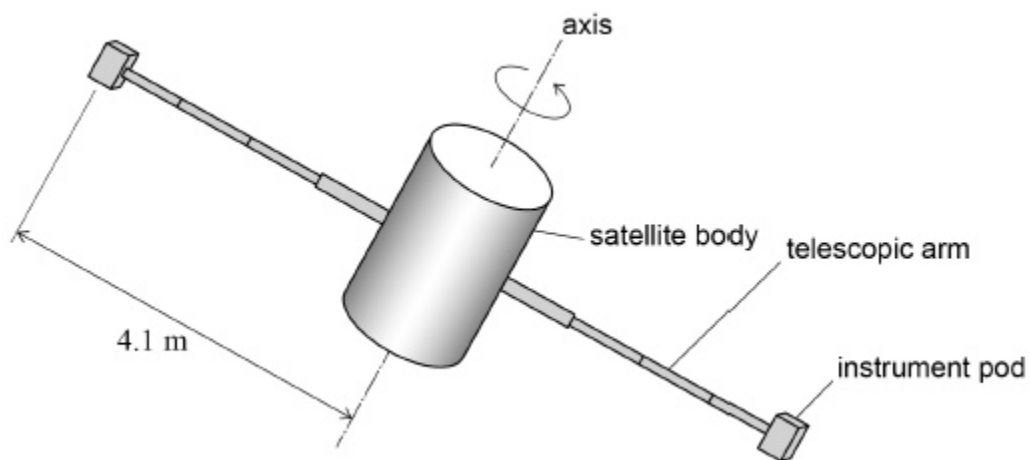
Compare the acceleration of the load in this experiment with its acceleration in the previous experiment.
A calculation is **not** required.

(3)
(Total 8 marks)

6. (a) State the law of conservation of angular momentum.

(1)

- (b) The diagram shows an orbiting satellite fitted with two small instrument pods attached to the ends of telescopic arms. The arms can be extended or retracted by a motor in the body of the satellite.



With the telescopic arms fully extended, the centre of mass of each instrument pod is at a radius of 4.1 m from the axis of rotation.

moment of inertia of satellite body about axis = 71 kg m^2

mass of each instrument pod = 5.0 kg

The mass of the telescopic arms is negligible.

Show that the total moment of inertia of the satellite with the arms fully extended is 240 kg m^2

(2)

- (c) The satellite is initially rotating slowly about its axis with the arms fully extended. The arms are slowly retracted so that the instrument pods move closer to the body of the satellite.

State and explain the change in the angular speed of the satellite as the arms are retracted.

(3)

- (d) The satellite is initially rotating at 1.3 rad s^{-1} with the telescopic arms fully extended. When fully retracted the instrument pods are at a radius of 0.74 m from the axis. The satellite contains sensitive equipment that may be damaged if the rotational speed exceeds 4.2 rad s^{-1}

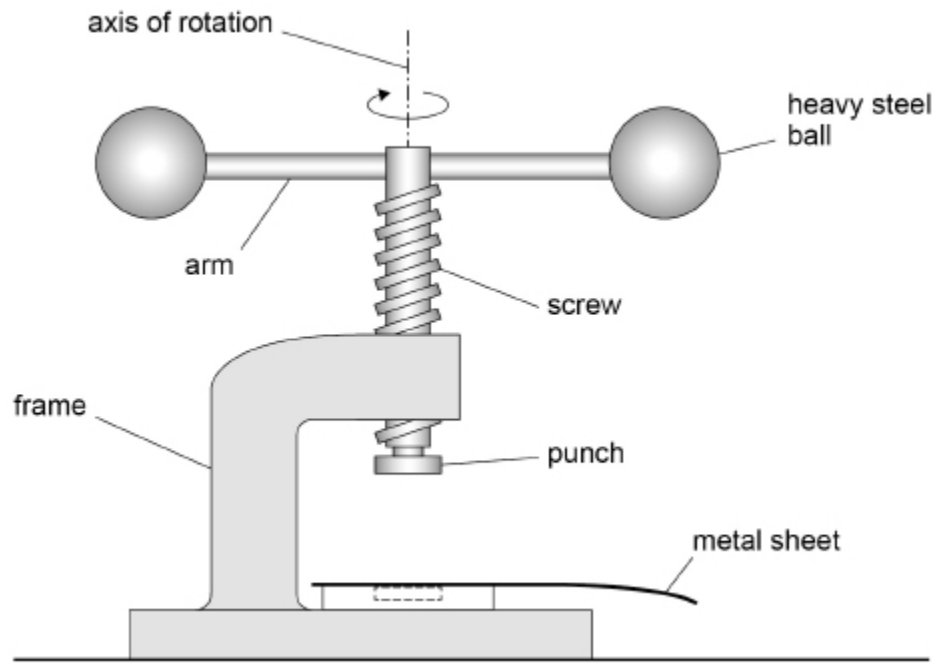
Deduce whether the arms can be retracted fully without the satellite exceeding its maximum permitted angular speed.

(3)

(Total 9 marks)

7.

The fly-press shown below is used by a jeweller to punch shapes out of a thin metal sheet.



The frame holds a screw and punch. Two arms are attached to the screw, each loaded with a heavy steel ball. The screw is driven downwards when the arms are rotated. Kinetic energy is stored in the rotating parts: the balls, arms, screw and punch. This energy is used to punch the shape out of the metal sheet.

- (a) When the punch reaches the metal sheet, the rotational speed of the arms is 2.9 rev s^{-1} . At this speed the rotational kinetic energy of the rotating parts is 10.3 J .

Calculate the moment of inertia of the rotating parts about the axis of rotation.

moment of inertia = _____ kg m^2

(2)

- (b) The total mass of the screw, punch and arms is the same as the total mass of the two balls.

Explain why the moment of inertia of the screw, punch and arms about the axis of rotation is **much** smaller than the moment of inertia of the steel balls about the same axis.

(2)

- (c) During the punching of the metal sheet, the rotating parts of the fly-press are brought uniformly to rest from an initial rotational speed of 2.9 rev s^{-1} in a time of 89 ms.

Determine

- the angular deceleration
- the angle turned through by the rotating parts.

angular deceleration = _____ rad s^{-2}

angle = _____ rad

(3)

- (d) For thicker or stiffer metal sheets the rotational kinetic energy at 2.9 rev s^{-1} is not enough to punch out the shape.

The distance from the axis of rotation to the centre of each ball is y .

The radius of each ball is R .

The stored energy can be increased by

either

- increasing y by 15% without changing R

or

- increasing R by 15% without changing y .

Deduce which of these would produce the greater increase in stored energy.

(3)

- (e) Which of the following is the SI unit for angular impulse?

Tick (✓) **one** box.

N m s^{-1}

N s

N m s

$\text{kg m}^2 \text{s}^{-2}$

(1)

(Total 11 marks)

8.

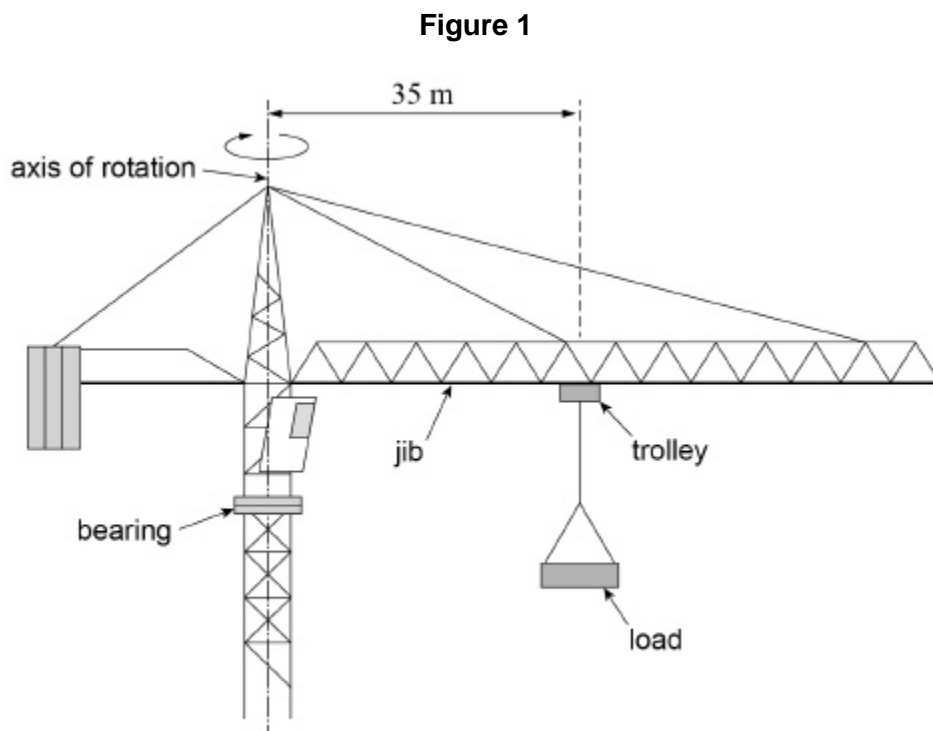
- (a) There is an analogy between quantities in rotational and translational dynamics.

Complete the table, stating in words the quantities in rotational dynamics that are analogous to force and mass in translational dynamics.

Translational dynamics	Rotational dynamics
force	
mass	

(2)

Figure 1 shows a side view of the jib of a tower crane. The load is supported by a trolley which can move along the jib. The jib consists of all the parts of the crane above the bearing, but excluding the trolley and load.



The moment of inertia of the jib about the axis of rotation = $2.6 \times 10^7 \text{ kg m}^2$

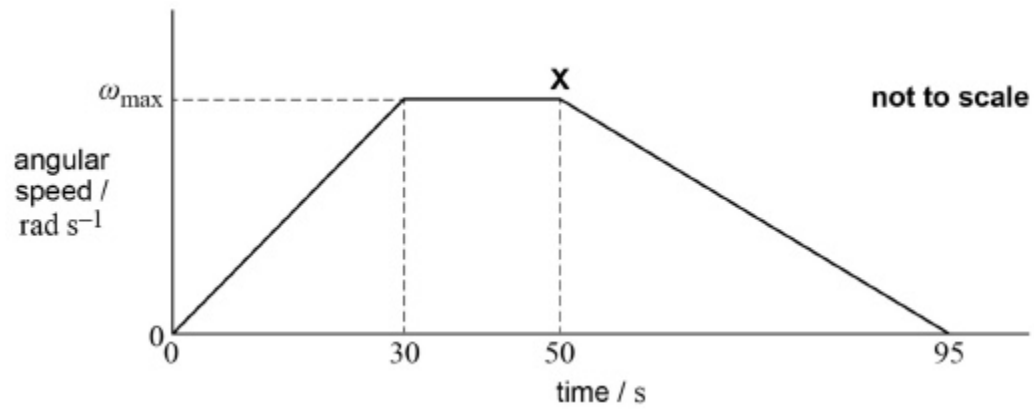
Mass of trolley and load = $2.2 \times 10^3 \text{ kg}$

(b) The load is at a distance of 35 m from the axis of rotation.

Show that the total moment of inertia of the jib, and the trolley and load, about the axis of rotation is about $3 \times 10^7 \text{ kg m}^2$.

- (c) **Figure 2** shows the variation of angular speed of the jib as it turns through an angle of 4.7 rad (270°) in a total time of 95 s. The trolley and load remain at a distance of 35 m from the axis.

Figure 2



Calculate the maximum angular speed ω_{\max} of the jib.

maximum angular speed = _____ rad s^{-1}

(2)

- (d) At time **X** in **Figure 2** the motor that is driving the jib is disengaged. A constant braking torque is then applied to bring the jib to a standstill from its maximum angular speed.

The crane driver repeats the movement of the jib with the same load at 35 m from the axis of rotation. Up to time **X** the motion is the same as before. From time **X** the trolley is driven at a steady speed away from the axis as the jib continues to rotate until the jib comes to a standstill.

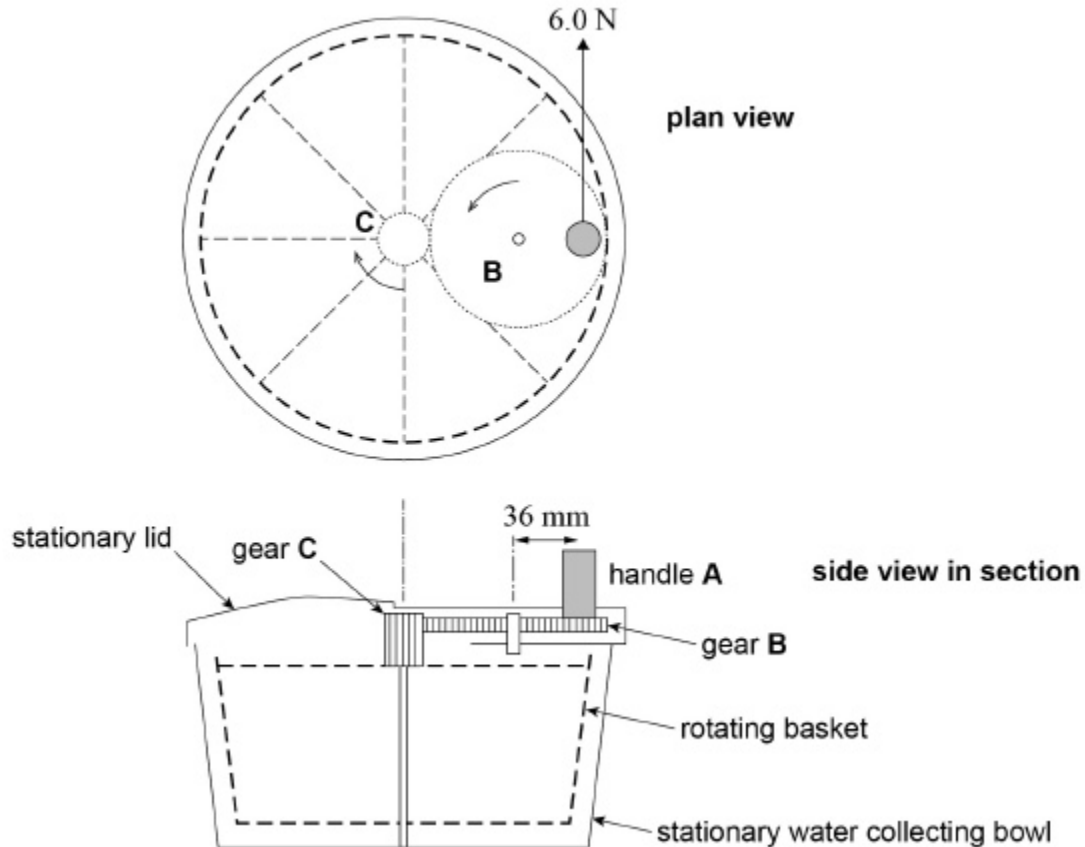
Assume the braking torque remains the same as before.

Discuss how the motion of the trolley affects the time taken for the jib to come to a standstill.

(3)

(Total 8 marks)

9. The diagram shows the basic principle of operation of a hand-operated salad spinner used to dry washed salads.



The salad is placed in the basket and the lid is attached.

When handle **A** is turned the basket and its contents spin rapidly. Water on the salad is driven through holes in the basket into the stationary water collecting bowl. The pivot for gear **B** is fixed to the lid. This pivot and the lid do not move. When gear **B** rotates, gear **C** also rotates but at a greater angular speed. Gear **C** is fixed to the basket and rotates it.

A force of 6.0 N is applied to handle **A** as shown. Handle **A** is at a radius of 36 mm from its centre of rotation.

- (a) Calculate the input torque.

torque = _____ N m

(1)

- (b) Gear **C** rotates four times for every one revolution of gear **B**.

Deduce whether it is possible for the torque on gear **C** to be greater than one quarter of the input torque.

(2)

- (c) It takes 2.1 s for the empty basket to reach an angular speed of 76 rad s^{-1} . The torque on gear **C** is a constant 0.054 N m during this time. Frictional losses are negligible.

Calculate the moment of inertia of the basket about its axis of rotation.

moment of inertia = _____ kg m^2

(2)

- (d) The gears are made from polymer (plastic). An early version of this salad spinner suffered from damaged gear teeth.

Explain with reference to angular impulse why a great force is put on the gear teeth if the user tries to stop the loaded basket too quickly using the handle.

(3)
(Total 8 marks)

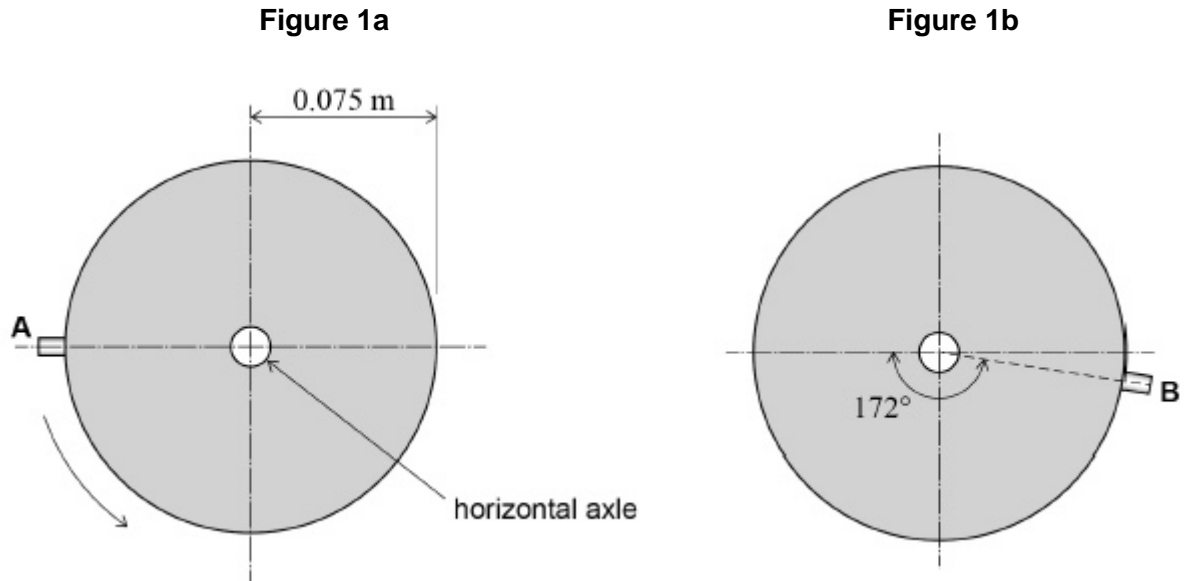
10.

- (a) State **one** function of a flywheel.

(1)

- (b) A student does an experiment to determine the frictional torque acting at the bearings of a steel flywheel. The flywheel has a radius of 0.075 m and is perfectly balanced.

The student places a small magnet of mass 0.020 kg at point **A** on the circumference of the flywheel on a horizontal line through the axis of rotation as shown in **Figure 1a**. The student releases the flywheel. The flywheel first comes to rest when it has moved through an angle of 3.00 rad (172°), with the magnet now in position **B** as shown in **Figure 1b**.



The loss in gravitational potential energy of the magnet equals the work done against the frictional torque acting at the bearings.

Show that the frictional torque is about 7×10^{-4} N m

- (c) The student goes on to determine the moment of inertia of the flywheel. The magnet is removed and the flywheel is made to spin. Measurements show that the flywheel makes 573 rotations as its angular speed reduces uniformly from 25.0 rad s^{-1} to zero. Assume the frictional torque at the bearings is constant and the same as in question (b).

Determine the moment of inertia of the flywheel about its axis of rotation.

moment of inertia = _____ kg m^2

(3)

(Total 7 marks)